FORMALISMO MATEMÁTICO EM MODELOS COSMOLÓGICO DA GRAVITAÇÃO

MATHEMATICAL FORMALISM IN COSMOLOGICAL MODELS OF GRAVITY

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Resumo

Analisamos soluções para cosmologias Friedmann-Robertson-Walker em teoria Brans-Dicke, onde um campo escalar é acoplado à gravidade. As soluções cosmológicas da teoria Brans-Dicke são caracterizadas por uma densidade de energia do vácuo em decomposição e por uma densidade de matéria relativa constante, considerandas no universo de Robertson-Walker, soluções exatas foram obtidos em Brans-Dicke cosmologia na ausência do termo cosmológico Λ. A matéria é governada pela γ-lei, por um fluido perfeito, incluindo a energia no vácuo como um caso especial. Através de uma mudança de variáveis, reduzimos as equações de campo de quarta ordem para a segunda ordem, e eles se tornam equivalente em um sistema dinâmico e com soluções. Nas propriedades de isotropia e homogeneidade, o princípio cosmológico exige que o conteúdo de matéria universal seja descrito como um fluido perfeito, para o qual se assume um tensor energia-momentum. Portanto, nosso objetivo é encontrar as equações que regem a dinâmica desse conteúdo em um universo inflacionário.

Palavras-chave: Lei dos Gases Perfeitos, Equações de Campo; Constante de acoplamento.

Abstract

We analyze solutions to Friedmann-Robertson-Walker cosmologies in Brans-Dicke theory, where a scalar field is coupled to gravity. The cosmological solutions of Brans-Dicke theory, characterized by a decaying vacuum energy density and by a constant relative matter density, considering Robertson-Walker universe, exact solutions have been obtained in Brans-Dicke cosmology in absence of term Λ cosmological. Matter is modelled by a γ-law perfect fluid, including vacuum energy as a special case. The properties of homogeneity and isotropy, the cosmological principle requires that the contents of universal matter are described as a perfect fluid, for which if an assumed energy-momentum tensor. Therefore, our goal is to find the equations that govern the dynamics of this content in an inflationary universe.

Key words: Perfect Gas law; Field Equations; Coupling constant.

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1. INTRODUCTION

Several authors have been considering the possibility of a varying cosmological term in order to fit the observed non decelerated expansion of the universe and, at the same time, to explain the small value of the cosmological constant observed at present, many authors have studied the problems with cosmological solutions involving time dependent cosmological term and Brans-Dicke field. Some of the researchers who have the contribution in this field (BERMAN; SOM; GOMIDE, 1989). There is considerable interest, therefore, both from inflationary cosmology and from string theory, to revisit cosmology in the scalar-tensor gravity of Brans and Dicke. Among all the problems we have considered more general solutions than those presented by Bertolami for a Robertson-Walker metric and perfect fluid obeying the perfect gas law of state (BERTOLAMI, 1986a). Bertolami considered in Brans-Dicke cosmology a time dependent cosmological term, having in mind that it should explain why the present value of Λ is 10^{50} times smaller than in the Glashow-Salam-Weinberg model, and 10^{107} times smaller than in a GUT theory (BERTOLAMI,1986b). Langacker, solved it for the Robertson-Walker metric with the cases $p = 0$ and $p = \frac{1}{3}\rho$, where p is the pressure and ρ is the density (LANGACKER, 1981). Among the various gravitational theories the only one still observationally consistent is Brans-Dicke theory. Cosmography is the study of the large-scale structure of spacetime. The goal of cosmography is to measure spacetime geometry or, in other words, to determine the spacetime metric. For obvious practical reasons astronomers cannot use clocks and measuring rods or even lasers to measure spacetime structure, making this a challenging endeavor that has yet to fully succeed. In this theory, the gravitational strength is made time dependent in order to incorporate Mach's principal into general relativity and is related with a scalar field through the relationship $G = a\phi^{-1}$ where 'a' is a positive constant given by,

$$a = \frac{4 + 2w}{3 + 2w} > 0$$

where w is the Brans-Dicke coupling constant, and the positive sign of 'a' is necessary in order that a given mass bend light in the correct direction. With this variation law for *G*, we will form an empirical approach to be used in Brans-Dicke equations. Observational tests of the Robertson-Walker family of cosmological models (or any others) are made by comparing predictions with observations of astronomically measurable quantities such as the redshift, age, angular size, flux (of electromagnetic radiation) and abundance of objects of various types (galaxies, quasars, etc.). Because of the non-Minkowski geometry of spacetime the dependence of these quantities on distance and even the meaning of distance itself are more complicated than in special relativity. In the present paper, the exact solutions can be obtained in Brans-Dicke cosmology in absence of cosmological term Λ by considering flat Robertson-Walker universe. We also consider a perfect fluid obeying the perfect gas law of state in the manner as applied by Berman (BERMAN, 1991a). In section 2 we have presented the field equations and their solutions. The physical and geometrical properties of the solutions obtained are also discussed in the last section.

2. FIELD EQUATIONS AND THEIR SOLUTIONS

The metric considered for the present problem is the Robertson–Walker metric

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1 - Kr^2} + r^2 d\theta + r^2 \sin^2\theta d\varphi^2\right\}, \quad (1)$$

where 't' is the cosmic time; R(t) , the scale factor of the universe, (r,θ,ϕ) the coordinate of co-moving observers at rest with respect to cosmic fluid and K is the curvature index which takes the values +1, 0 and –1 for a closed, flat and open model of the universe respectively. The properties of homogeneity and isotropy, the cosmological principle requires that the content of universal matter is described as a perfect fluid, for which if an assumed energy-momentum tensor.

The energy momentum tensor $T_{\mu\nu}$, for the perfect fluid is given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \qquad (2)$$

where ρ is the mass density, p is the isotropic pressure (WEINBERG,1989).

Corresponding to the metric (1), the four velocity vector satisfies the equation

$$g_{\mu\nu} U_\mu U_\nu = 1, \qquad (3)$$

considering the co-moving co-ordinate system, we obtain

$$U_1 = U_2 = U_3 = 0 \text{ and } U_4 = 1, \qquad (4)$$

Bertolami (BERMAN, 1991b) equations are

$$\frac{\ddot{\phi}}{\phi} + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} + \frac{\Lambda}{3} - \frac{\phi}{3}\frac{\partial\Lambda}{\partial\phi} = \frac{8\pi a}{(3+2w)\phi}(\rho - 3p), \qquad (5)$$

$$\frac{\dot{R}^2}{R^2} + \frac{K}{R^2} = \frac{8\pi a\rho}{3\phi} - \frac{\dot{\phi}}{\phi}\frac{\dot{R}}{R} + \frac{w}{6}\frac{\dot{\phi}^2}{\phi^2} - \frac{2}{3}\Lambda + \frac{\phi}{3}\frac{\partial\Lambda}{\partial\phi}, \qquad (6)$$

$$\frac{-8\pi a}{\phi^2}\dot{\phi}\rho + \left(w\frac{\dot{\phi}}{\phi} - 3\frac{\dot{R}}{R}\right)\frac{\ddot{\phi}\phi - \dot{\phi}^2}{\phi^2} - \frac{3}{R^2}\left(\ddot{R}R - \dot{R}^2\right)\frac{\dot{\phi}}{\phi} = \frac{\partial\Lambda}{\partial\phi}\dot{\phi} - 3\frac{\dot{R}}{R}\left(\frac{8\pi a}{\phi}\rho + \frac{w}{2}.\frac{\dot{\phi}^2}{\phi^2} - 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \Lambda\right) \qquad (7)$$

$$\dot{\rho} = -\frac{3\dot{R}}{R}(\rho + p), \qquad (8)$$

For flat model of the universe in absence of the cosmological term Bertolami's equations become

$$\frac{\ddot{\phi}}{\phi} + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} = \frac{8\pi a}{(3+2w)\phi}(\rho - 3p), \qquad (9)$$

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi a\rho}{3\phi} - \frac{\dot{\phi}}{\phi}\frac{\dot{R}}{R} + \frac{w}{6}\frac{\dot{\phi}^2}{\phi^2}, \qquad (10)$$

$$\frac{-8\pi a}{\phi^2}\dot{\phi}\rho + \left(w\frac{\dot{\phi}}{\phi} - 3\frac{\dot{R}}{R}\right)\frac{\ddot{\phi}\phi - \dot{\phi}^2}{\phi^2} - \frac{3}{R^2}\left(\ddot{R}R - \dot{R}^2\right)\frac{\dot{\phi}}{\phi} = -3\frac{\dot{R}}{R}\left(\frac{8\pi a}{\phi}\rho + \frac{w}{2}.\frac{\dot{\phi}^2}{\phi^2} - 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}\right), \qquad (11)$$

and

$$\dot{\rho} = -\frac{3\dot{R}}{R}(\rho + p), \qquad (12)$$

Now, let us assume certain physical relations $\frac{\dot{R}}{R} = H$, where H is the Hubble's constant i.e.

$$R = Ae^{Ht}, \qquad (13)$$

and

$$\phi = BR^\gamma, \qquad (14)$$

From equations (13) and (14), we get

$$\phi = BA^{\gamma} e^{H\gamma t}, \tag{15}$$

Using the equation of state $p = \in \rho$ in equation (8), we get

$$\rho = A_1 e^{-3H(1+\in)t}, \tag{16}$$

where A_1 is a constant. By using the above equation of state and equation (14) in equation (9), we obtain

$$\gamma^2 H^2 + 3\gamma H^2 = \frac{8\pi a}{3+2w} A^{-\gamma} B^{-1} A_1 (1 - 3\in) e^{-\{\gamma + 3(1+\in)\}Ht}, \tag{17}$$

So the coupling constant "w" will be independent of time 't' if

$$-\{\gamma + 3(1+\in)\}H = 0, \tag{18}$$

For the expanding model of the universe, $H = \frac{\dot{R}}{R} \neq 0$

$$\therefore \gamma + 3(1+\in) = 0, \tag{19}$$

By the use of the relation (19) the above equation (17) reduces to

$$(3+2w)A^{\gamma} B(3+\gamma)\gamma H^2 = 8\pi a A_1 (1 - 3\in), \tag{20}$$

Now from equation (10), we get

$$H^2 + \gamma H^2 - \frac{w}{6}\gamma^2 H^2 = \frac{8\pi a A_1}{3A^{\gamma} B}, \tag{21}$$

Using equation (13), (14) and (16) the above equation (11) becomes

$$8\pi a A_1 (3 - \gamma) = 3A^{\gamma} H^2 B\gamma \left(3 - \frac{w}{2}\gamma\right), \tag{22}$$

From the equation of state $p = \in \rho$, we get

$$p = \{A_1 e^{-3H(1+\in)t}\}. \in, \tag{23}$$

From equation (15), we get

$$\phi = B.\frac{1}{(Ae^{Ht})^{3(1+\in)}}, \tag{24}$$

3. PHYSICAL INTERPRETATION OF THE SOLUTIONS

From equation (19) and (15) we can conclude that the Brans-Dicke scalar ϕ is decreasing function of R since γ is found to be negative. In this sense, the Brans-Dicke theory we are using here, with constant "ω", must be considered an effective description, valid only in the cosmological limit. A more general approach can be based on scalar-tensor theories in which "ω" depend on the scale, being very high in the weak field approximation of Solar System. From equation (15) we further observe that the B-D scalar ϕ is found to be exponentially decreasing function of time. From equation (16) the mass density ρ is also an exponentially decreasing function of time for all distributions i.e. dust distribution (p=0), stiff fluid distribution (p=ρ) and disordered distribution of radiation (p=3p). From equation (13) and (16) we obtain the relation $\rho = C'\phi$, where $C' = \dfrac{A_1}{BA^{\gamma}}$ is an arbitrary constant (SAHA,2006a). It implies that $p \infty \phi$. The deceleration parameter, $q = -\dfrac{R\ddot{R}}{\dot{R}^2} = -1$. At time t = 0, the mass density ρ becomes a constant and it tends to zero as $t \to \infty$. Similarly, the Brans-Dicke scalar ϕ reduces to a constant at time t = 0 and it tends to zero as $t \to \infty$.

Case I: When $\in = 0$

We have $\gamma = -3$ and two different values of "w" i.e. $w = -\dfrac{4}{3}$ and -2.

This is something absurd. Therefore we cannot take the value of $\in = 0$ i.e. dust distribution cannot interact with Brans-Dicke scalar ϕ.

From equation (19), we obtain $\gamma = -3$ when $\in = 0$.

From equation (20), we obtained $A_1 = 0$ $\sin ce\ a \neq 0$ by the relation $a = \dfrac{4+2w}{3+2w} > 0.$ Therefore the mass density ρ is found to be zero. It implies that there is no interaction between B-D scalar ϕ and perfect fluid when $\in = 0$.

Case II: When $\in = 1$

We have $\gamma = -6$ and two different values of "w" i.e. $w = -\dfrac{7}{4}$ and $-\dfrac{3}{4}$.

So the in the case of the stiff fluid distribution also there is no interaction between the B-D scalar ϕ and perfect fluid.

Case III: When $\in = \dfrac{1}{3}$

We find that $\gamma = -4$ and two different values "w" i.e. $w = -\dfrac{3}{2}$ and $-\dfrac{9}{8}$.

Also $A_1 = 0$. It means that in the case of disordered distribution of radiation, ϕ. The distribution cannot interact with the Brans-Dicke scalar so there is no interaction between B-D scalars ϕ and perfect fluid since

$\rho = 0$. Hence, we cannot derive the solutions of the particular cases corresponding to $\in = 0, -1, \frac{1}{3}$ since we obtain two different values of the coupling parameter "w" in each case.

CONCLUSIONS

- Most cosmological tests rely, directly or indirectly, on the angular part of the Robertson-Walker metric and not only on the radial part used above;
- As a result, one may make use of the framework of General Relativity, namely, the Friedmann equations and Robertson-Walker metric, and the analysis turns out simplified;
- flat Friedmann-Robertson-Walker Universe, and we will show how it is possible to write a modification to gravity in the fluid;
- On the other hand, the observed small value of the Cosmological Constant leads to several conceptual problems (vacuum energy, coincidence problem, etc.), so that in the last few years, several different approaches to the dark energy issue have been proposed. Among them, the modified theories of gravity represent an interesting extension of Einstein's theory, but also supersymmetry and string theories have been investigated.

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